

## Review

# Design and performance optimization of GPU-3 Stirling engines

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Received 30 May 2007

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**Abstract**

To increase the performance of Stirling engines and analyze their operations, a second-order Stirling model, which includes thermal losses, has been developed and used to optimize the performance and design parameters of the engine. This model has been tested using the experimental data obtained from the General Motor GPU-3 Stirling engine prototype. The model has also been used to investigate the effect of the geometrical and physical parameters on Stirling engine performance and to determine the optimal parameters for acceptable operational gas pressure. When the optimal design parameters are introduced in the model, the engine efficiency increases from 39% to 51%; the engine power is enhanced by approximately 20%, whereas the engine average pressure increases slightly.

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**Keywords:** Stirling engine; Design; Dynamic model; Losses; Regenerator; Power; Thermal efficiency

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**1. Introduction**

An elementary Stirling engine is composed of an engine piston, an exchanger piston, and three heat exchangers: a cooler, a regenerator, and a heater. The exchanger and the engine pistons are connected by mechanical transmission,

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**Nomenclature**

|               |                                                             |
|---------------|-------------------------------------------------------------|
| $A$           | area (m <sup>2</sup> )                                      |
| $C_p$         | specific heat at constant pressure (J/(kg K))               |
| $C_{pr}$      | heat capacity of each cell matrix (W/K)                     |
| $\varepsilon$ | regenerator efficiency                                      |
| $h$           | convection heat transfer coefficient (W/(m <sup>2</sup> K)) |
| $M$           | mass of working gas in the engine (kg)                      |
| $\dot{m}$     | mass flow rate (kg/s)                                       |
| $m$           | mass of gas in different components (kg)                    |
| $P$           | pressure (Pa)                                               |
| $Q$           | heat (J)                                                    |
| $\dot{Q}$     | power (W)                                                   |
| $R$           | gas constant (J/(kg K))                                     |
| $T$           | temperature (K)                                             |
| $k$           | wall conductivity (W/(m K))                                 |
| $V$           | volume (m <sup>3</sup> )                                    |
| $W$           | work (J)                                                    |

$\omega$  operating frequency (rd/s)

**Subscripts**

|      |                   |
|------|-------------------|
| c    | compression space |
| d    | expansion space   |
| diss | dissipation       |
| E    | entered           |
| f    | cooler            |
| h    | heater            |
| irr  | irreversible      |
| P    | loss              |
| Pa   | wall              |
| r    | regenerator       |
| S    | outlet            |
| shtl | shuttle           |
| T    | total             |

as shown in Fig. 1. The engine uses external combustion, hence it can be powered by any source of energy (combustion energy, solar energy, etc.) and causes less pollution than the traditional engines [1–5]. The working piston converts gas pressure into mechanical power, whereas the exchanger piston is used to move gas between hot and cold working spaces. The engine presents an excellent theoretical efficiency of the same order as the Carnot efficiency.

Several prototypes have already been produced [2–5], but their actual efficiency remains very low compared to

the high theoretical efficiency. In fact, Stirling engines involve extremely complex phenomena related to compressible fluid mechanics, thermodynamics, and heat transfer. An accurate description and understanding of these highly non-stationary phenomena is necessary so that different engine losses and optimal design parameters may be determined.

Several authors have studied the finite-time thermodynamic performance of Stirling engines and the effect of heat losses and irreversibilities on engine performance [6–11]. However, they have not calculated the optimal design parameters for maximum power and efficiency. Popescu et al. [6] have shown that the low performance is mainly due to the non-adiabatic regenerator. Kaushic, Wu, and co-workers [7,13,15] have proved that the most important factors affecting the performance of Stirling engines are heat conductance between the engine and reservoirs, the imperfect regenerator coefficient and the rates of the regenerating process. Kongtragool and Wongwises [9] have investigated the effect of regenerator efficiency and dead volume on the engine network, heat input, and engine efficiency, using a theoretical approach to the thermodynamic analysis of Stirling engines. Costea et al. [10] have studied the effect of irreversibility on solar Stirling engine cycle performance. They have included the effect of incomplete heat regeneration, and internal and external irreversibility of the cycle. Cinar et al. [12] manufactured a beta-type Stirling engine operating at atmospheric pressure. The tests carried out on this engine have shown that the engine speed, engine torque, and power output increase with the hot source temperature. Walker [2] has identified several other losses, such as conduction losses in the exchangers, dissipation by pressure drop, shuttle and gas spring hysteresis losses. These losses, however, are not usually accounted for in the published work due to their complexity. Urieli and Berchowitz [14] have developed a quasi-steady flow model that includes

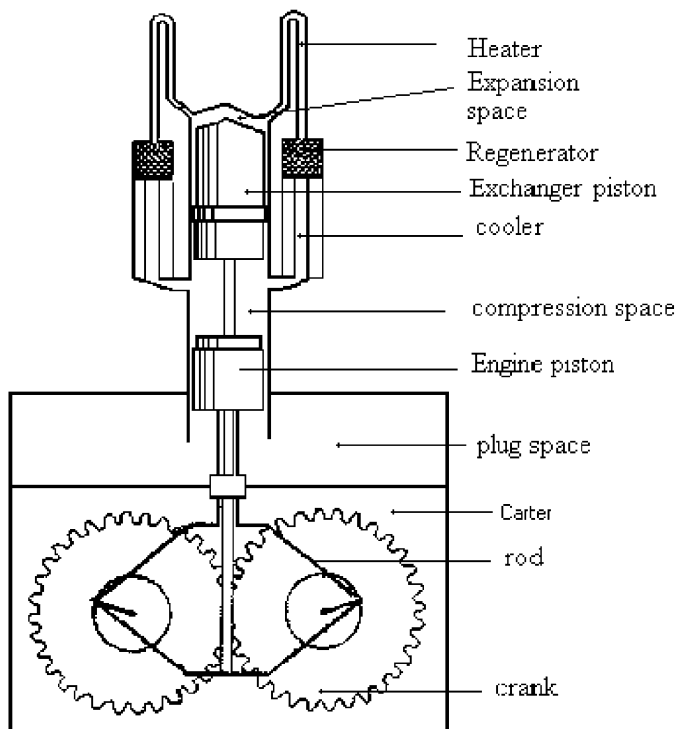


Fig. 1. Rhombic Stirling engine GPU-3 (built by General Motor [14]).

only the pressure drop in the exchangers. The results obtained by the author using this model are more accurate than those obtained by other models, though they are different from corresponding experimental data. Abdullah et al. [16] have given the design considerations to be taken into account when designing a low-temperature differential double-acting Stirling engine for solar applications. They have determined the optimal design configuration, such as the engine speed, regenerator porosity, and heat exchanger volumes, including the pressure drop, heat exchanger effectiveness, and the swept volume. Therefore, the Stirling engine performance depends on geometrical and physical parameters of the engine and the working fluid, such as regenerator efficiency, porosity, dead volume, swept volume, temperature of sources, energy and shuttle losses, etc.

A numerical simulation that accounts for losses has been developed by the authors and tested using the General Motor GPU-3 Stirling engine data [17]. The results obtained proved better than those obtained by other models and correlates more closely with the corresponding experimental data. The model is used to determine losses in different engine compartments and to calculate the geometrical and physical parameters corresponding to minimal losses [18,19]. An optimization based on this model is presented in this article. This study helps to determine the influence of geometrical and physical parameters on prototype performance and therefore to identify the optimal design parameters.

## 2. Dynamic model with losses

A second-order adiabatic model has been initially developed; the estimated engine parameters are computed for use in the validation. The results are compared with those obtained by Urieli and Berchowitz [14] at the same conditions (adiabatic model). Afterward, a dynamic model including losses in different engine elements has been developed.

The losses considered in this model include energy dissipation by pressure drops and internal conduction through exchangers. Furthermore, the following energy losses have also been included: the external conduction in the regenerator, the shuttle effect in the displacer, and the gas spring hysteresis in the compression and expansion spaces. However, mechanical friction between moving parts has not been considered.

### 2.1. Losses included in the model

The losses considered in the model and evaluated in articles [17,18], are:

1. energy dissipation by pressure drops in heat exchangers ( $\delta\dot{Q}_{\text{diss}}$ ), given by [14]

$$\delta\dot{Q}_{\text{diss}} = -\frac{\Delta p \dot{m}}{\rho} \quad (1)$$

where  $\Delta p$ , the frictional drag force, is given by

$$\Delta p = -\frac{2f_r \mu G V}{A d^2 \rho} \quad (2)$$

where  $G$  the working gas mass flow ( $\text{kg m}^{-2} \text{s}^{-1}$ ),  $d$  the hydraulic diameter,  $\rho$  the gas density ( $\text{kg m}^{-3}$ ),  $V$  the volume ( $\text{m}^3$ ), and  $f_r$  the Reynolds friction factor;

2. energy loss due to internal conduction ( $\delta\dot{Q}_{\text{pcd}}$ ) between the hot parts and the cold ones of the engine through the different exchangers [14]:

$$\delta\dot{Q}_{\text{Pcd r1}} = k_{\text{cd r1}} \frac{A_{\text{r1}}}{L_{\text{r}}} (T_{\text{r-r}} - T_{\text{f-r}}) \quad (3)$$

$$\delta\dot{Q}_{\text{Pcd r2}} = k_{\text{cd r2}} \frac{A_{\text{r}}}{L_{\text{r}}} (T_{\text{r-h}} - T_{\text{r-r}}) \quad (4)$$

$$\delta\dot{Q}_{\text{Pcd f}} = k_{\text{cd f}} \frac{A_{\text{f}}}{L_{\text{f}}} (T_{\text{f-r}} - T_{\text{c-f}}) \quad (5)$$

$$\delta\dot{Q}_{\text{Pcd h}} = k_{\text{cd h}} \frac{A_{\text{h}}}{L_{\text{h}}} (T_{\text{h-d}} - T_{\text{r-h}}) \quad (6)$$

where  $k_{\text{cd}}$  ( $\text{W m}^{-1} \text{K}^{-1}$ ) is the thermal conductance of the material and  $A$  the effective area for conduction;

3. energy loss due to external conduction ( $\delta\dot{Q}_{\text{pext}}$ ) in the regenerator which is not adiabatic. These losses are specified by the regenerator adiabatic coefficient  $\varepsilon \leq 1$ , defined as the ratio between the heat given up in the regenerator by the working gas during its transition toward the compression space and the heat received in the regenerator by the working gas during its transition toward the expansion space [3]. Hence the energy stored by the regenerator at the time of the transition of the gas from the expansion space to the compression space is not completely restored with this gas at the time of its return. For the ideal case of the regenerator with perfect insulation,  $\varepsilon = 1$ . The energy lost by external conduction is

$$\delta\dot{Q}_{\text{Pext}} = (1 - \varepsilon)(\delta\dot{Q}_{\text{r1}} + \delta\dot{Q}_{\text{r2}}) \quad (7)$$

The effectiveness of the regenerator  $\varepsilon$  is given starting from the equation below [8,14]:

$$\varepsilon = \frac{\text{NTU}}{1 + \text{NTU}} \quad (8)$$

NTU is the number of transfer units:

$$\text{NTU} = \frac{h A_{\text{wg}}}{C_p \dot{m}} \quad (9)$$

where  $h$  is the overall heat transfer coefficient (hot stream/matrix/cold stream),  $A_{\text{wg}}$  refers to the wall/gas, or “wetted” area of the heat exchanger surface,  $C_p$  the specific heat capacity at constant pressure, and  $\dot{m}$  the mass flow rate through the regenerator.

4. energy loss due to shuttle effect ( $\delta\dot{Q}_{\text{pshtl}}$ ): the displacer absorbs a quantity of heat from the hot source and

restores it to the cold one. This loss of energy is given by [16]

$$\delta \dot{Q}_{P_{shl}} = \frac{0.4Z^2 k_{pis} D_d}{J L_d} (T_d - T_c) \quad (10)$$

where  $J$  is the annular gap between the displacer and the cylinder (m),  $k_{pis}$  the piston thermal conductivity ( $\text{W m}^{-1} \text{K}^{-1}$ ),  $D_d$  the displacer diameter (m),  $L_d$  the displacer length (m),  $Z$  the displacer stroke (m), and  $T_d$  and  $T_c$  are, respectively, the temperature in the expansion space and in the compression space (K);

5. energy loss due to gas spring hysteresis ( $\delta \dot{Q}_{P_{irr}}$ ): For an ideal gas, the pressure/volume relationship is either isothermal or adiabatic. In a real gas, there is a certain amount of work that is dissipated ( $\delta \dot{Q}_{P_{irr}} = -\delta \dot{W}_{irr}$ ). Urieli and Berchowitz [14] gave the expression of this loss. For the compression space we have

$$\delta \dot{W}_{P_{irr c}} \cong \sqrt{\frac{1}{32} \omega \gamma^3 (\gamma - 1) T_{Pa c} P_{c moy} k_{Pa c}} \left( \frac{\Delta V_c}{V_{c moy}} \right)^2 A_{Pa c} \quad (11)$$

For the expansion space we have

$$\delta \dot{W}_{P_{irr d}} \cong \sqrt{\frac{1}{32} \omega \gamma^3 (\gamma - 1) T_{Pa d} P_{d moy} k_{Pa d}} \left( \frac{\Delta V_d}{V_{d moy}} \right)^2 A_{Pa d} \quad (12)$$

where  $\gamma = C_p/C_v$ ,  $T_{Pa c}$  and  $T_{Pa d}$  are the wall temperatures in the compression space and the expansion space, respectively, which are equal to the average temperature in these spaces;  $\Delta V_c$  and  $\Delta V_d$  are, respectively, the volume amplitude in the compression space and the expansion space,  $A_{Pa c}$  and  $A_{Pa d}$  are the wetted area in the compression space and the expansion space, respectively, and  $\omega$  is the operating frequency.

## 2.2. Model development

The schematic model of the engine and various temperature distributions in the engine components are shown in Fig. 2. The dynamic model of the developed Stirling engine is based on the following assumptions:

- (1) the gas temperature in the different engine elements varies linearly;
- (2) the cooler and the heater walls are maintained isothermally at temperatures  $T_{Pa f}$  and  $T_{Pa h}$ ;
- (3) the gas temperature in the different components is calculated using the perfect gas law:

$$T_c = \frac{P_c V_c}{R m_c} \quad (13)$$

$$T_f = \frac{P_f V_f}{R m_f} \quad (14)$$

$$T_h = \frac{P_h V_h}{R m_h} \quad (15)$$

$$T_d = \frac{P_d V_d}{R m_d} \quad (16)$$

- (4) the regenerator is divided into two cells r1 and r2; each cell has been associated with its respective mixed mean gas temperature  $T_{r1}$  and  $T_{r2}$  expressed as follows:

$$T_{r1} = \frac{P_{r1} V_{r1}}{R m_{r1}} \quad (17)$$

$$T_{r2} = \frac{P_{r2} V_{r2}}{R m_{r2}} \quad (18)$$

An extrapolated linear curve is drawn through temperature values  $T_{r1}$  and  $T_{r2}$ , defining the regenerator interface temperature  $T_{f-r}$ ,  $T_{r-r}$ , and  $T_{r-h}$ , as follows [18]:

$$T_{f-r} = \frac{3T_{r1} - T_{r2}}{2} \quad (19)$$

$$T_{r-r} = \frac{T_{r1} + T_{r2}}{2} \quad (20)$$

$$T_{r-h} = \frac{3T_{r2} - T_{r1}}{2} \quad (21)$$

According to the flow direction of the fluid, the interface's temperatures shown in Fig. 2 are defined as follows [17]:

$T_{c-f}$ , the temperature of the interface between the compression space and the cooler, is  $T_{c-f} = T_c$  if  $\dot{m}_{c-f} > 0$ , otherwise  $T_{c-f} = T_f$ .

$T_{f-r}$ , the temperature of the interface between the cooler and the regenerator, is  $T_{f-r} = T_f$  if  $\dot{m}_{f-r} > 0$ , otherwise  $T_{f-r} = T_{r-f}$ .

$T_{r-h}$ , the temperature of the interface between the regenerator and the heater, is  $T_{r-h} = T_{r-r}$  if  $\dot{m}_{r-h} > 0$ , otherwise  $T_{r-h} = T_h$ .

$T_{h-d}$ , the temperature of the interface between the heater and the expansion space, is  $T_{h-d} = T_h$  if  $\dot{m}_{h-d} > 0$ , otherwise  $T_{h-d} = T_d$ .

Heat is transferred to the working gas by means of forced convection given by

$$\delta \dot{Q} = h A_{Pa} (T_{Pa} - T_f) \quad (22)$$

where  $h$  is the heat transfer coefficient. Taking into account the losses by internal conduction in the exchangers and external conduction in the regenerator, the power exchanged in the different heat exchangers are given by

$$\delta \dot{Q}_f = h_f A_{Pa f} (T_{Pa f} - T_f) - \delta \dot{Q}_{P_{c d f}} \quad (23)$$

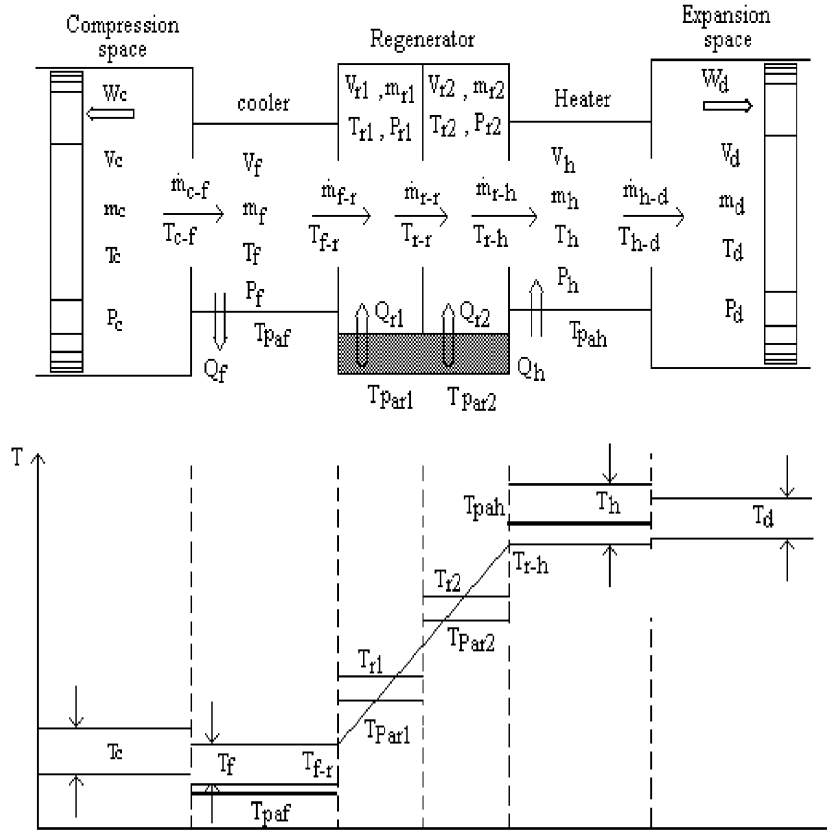


Fig. 2. Schematic model of the engine and various temperature distributions.

$$\delta \dot{Q}_{r1} = \varepsilon h_{r1} A_{Pa r1} (T_{Pa r1} - T_{r1}) - \frac{\delta \dot{Q}_{Pc d r1}}{2} \quad (24)$$

$$\delta \dot{Q}_{r2} = \varepsilon h_{r2} A_{Pa r2} (T_{Pa r2} - T_{r2}) - \frac{\delta \dot{Q}_{Pc d r2}}{2} \quad (25)$$

$$\delta \dot{Q}_h = h_h A_{Pa h} (T_{Pa h} - T_h) - \delta \dot{Q}_{Pc d h} \quad (26)$$

where  $\varepsilon$  is the regenerator efficiency.

The heat transfer coefficient of exchanges,  $h_f$ ,  $h_{r1}$ ,  $h_{r2}$ , and  $h_h$ , are available only empirically, being complicated functions of the fluid transport properties, the flow regime, and the heat exchanger geometry [14].

The regenerator matrix temperatures are, therefore, given by

$$\frac{dT_{Pa r1}}{dt} = -\frac{\delta \dot{Q}_{r1}}{C_{Pr} dt} \quad (27)$$

$$\frac{dT_{Pa r2}}{dt} = -\frac{\delta \dot{Q}_{r2}}{C_{Pr} dt} \quad (28)$$

There is no leakage, the total mass of gas in the system ( $M$ ) being constant. Thus

$$M = m_c + m_f + m_{r1} + m_{r2} + m_h + m_d \quad (29)$$

The energy equation applied to a generalized cell is reproduced as follows:

$$\delta \dot{Q} + C_p T_E \dot{m}_E - C_p T_S \dot{m}_S = \frac{d}{dt} W + C_v \frac{d(mT)}{dt} \quad (30)$$

Applying the energy equation to the compression space, we obtain

$$\delta \dot{Q}_c - C_p T_{c-f} \dot{m}_{c-f} = \frac{d}{dt} W_c + C_v \frac{d}{dt} (m_c T_c) \quad (31)$$

Since the compression space is adiabatic,  $\delta \dot{Q}_c = 0$  and the work done is  $dW_c/dt = P dV_c/dt$ . From continuity considerations, the rate of accumulation of gas ( $\dot{m}_c$ ) is equal to the mass inflow of gas, given by  $-\dot{m}_{c-f}$ . Thus, Eq. (31) reduces to

$$C_p T_{c-f} \dot{m}_c = P \frac{d}{dt} V_c + C_v \frac{d}{dt} (m_c T_c) \quad (32)$$

Substituting the equation of state and the associated ideal gas relations  $C_p - C_v = R$ ,  $C_p = R\gamma/\gamma - 1$ , and  $C_v = R/\gamma - 1$  into Eq. (32) and simplifying gives

$$\dot{m}_c = \frac{1}{RT_{c-f}} \left( P \frac{dV_c}{dt} + \frac{V_c dP}{\gamma dt} \right) \quad (33)$$

Taking into account the loss of gas spring hysteresis in the compression and expansion space,  $\delta W_{irr c}/dt$  and  $\delta W_{irr d}/dt$ , the work generated by the cycle can be expressed as

$$\frac{\delta W}{dt} = P_c \frac{dV_c}{dt} + P_d \frac{dV_d}{dt} - \frac{\delta W_{irr c}}{dt} - \frac{\delta W_{irr d}}{dt} \quad (34)$$

The total engine volume is

$$V_T = V_c + V_f + V_{r1} + V_{r2} + V_h + V_d \quad (35)$$

Since there is a variable pressure distribution throughout the engine, we have arbitrarily chosen the compression space pressure  $P_c$  as the baseline pressure. For each increment of the solution,  $P_c$  is evaluated from the relevant differential equation and the pressure distribution is determined with respect to  $P_c$ . Thus, it can be obtained from the following expression:

$$P_f = P_c + \frac{\Delta P_f}{2} \quad (36)$$

$$P_{r1} = P_f + \frac{(\Delta P_f + \Delta P_{r1})}{2} \quad (37)$$

$$P_{r2} = P_{r1} + \frac{(\Delta P_{r1} + \Delta P_{r2})}{2} \quad (38)$$

$$P_h = P_{r2} + \frac{(\Delta P_{r1} + \Delta P_h)}{2} \quad (39)$$

$$P_d = P_h + \frac{\Delta P_h}{2} \quad (40)$$

The other variables of the dynamic model with losses are given by the energy and mass conservation equation applied to a generalized cell. Taking into account energy dissipation caused by pressure drop in the exchangers ( $\delta \dot{Q}_{diss}$ ) and the other losses yields

$$\begin{aligned} \delta \dot{Q} - \delta \dot{Q}_{diss} - \delta \dot{Q}_{P_{shtl}} + C_p T_E \dot{m}_E - C_p T_S \dot{m}_S \\ = \frac{dW}{dt} - \frac{dW_{irr}}{dt} + C_V \frac{d(mT)}{dt} \end{aligned} \quad (41)$$

Substituting the ideal gas relations into Eq. (41) and simplifying gives

$$\begin{aligned} \delta \dot{Q} - \delta \dot{Q}_{diss} - \delta \dot{Q}_{P_{shtl}} + C_p T_E \dot{m}_E - C_p T_S \dot{m}_S \\ = \frac{1}{R} \left( C_p P \frac{dV}{dt} + C_V V \frac{dP}{dt} \right) - \delta \dot{W}_{irr} \end{aligned} \quad (42)$$

Applying expanded energy conservation Eq. (42) to the different engine cells in Fig. 2 gives

$$-C_p T_{c-f} \dot{m}_{c-f} = \frac{1}{R} \left( C_p P_c \frac{dV_c}{dt} + C_V V_c \frac{dP_c}{dt} \right) - \delta \dot{W}_{irr c} \quad (43)$$

$$\delta \dot{Q}_f - \delta \dot{Q}_{diss f} + C_p T_{c-f} \dot{m}_{c-f} - C_p T_{f-r} \dot{m}_{f-r} = \frac{C_V V_f}{R} \frac{dP_c}{dt} \quad (44)$$

$$\delta \dot{Q}_{r1} - \delta \dot{Q}_{diss r1} + C_p T_{f-r} \dot{m}_{f-r} - C_p T_{r-r} \dot{m}_{r-r} = \frac{C_V V_{r1}}{R} \frac{dP_c}{dt} \quad (45)$$

$$\delta \dot{Q}_{r2} - \delta \dot{Q}_{diss r2} + C_p T_{r-r} \dot{m}_{r-r} - C_p T_{r-h} \dot{m}_{r-h} = \frac{C_V V_{r2}}{R} \frac{dP_c}{dt} \quad (46)$$

$$\delta \dot{Q}_h - \delta \dot{Q}_{diss h} + C_p T_{r-h} \dot{m}_{r-h} - C_p T_{h-d} \dot{m}_{h-d} = \frac{C_V V_h}{R} \frac{dP_c}{dt} \quad (47)$$

$$C_p T_{h-d} \dot{m}_{h-d} - \delta \dot{Q}_{P_{shtl}} = \frac{1}{R} \left( C_p P_d \frac{dV_d}{dt} + C_V V_d \frac{dP_c}{dt} \right) - \delta \dot{W}_{irr d} \quad (48)$$

Summing Eqs. (43)–(48), we obtain the pressure variation:

$$\frac{dP_c}{dt} = \frac{1}{C_V V_T} \left[ R(\delta \dot{Q} - \delta \dot{Q}_{diss T}) - C_p \frac{\delta W}{dt} \right] \quad (49)$$

where  $\delta \dot{Q} = \delta \dot{Q}_f + \delta \dot{Q}_{r1} + \delta \dot{Q}_{r2} + \delta \dot{Q}_h - \delta \dot{Q}_{P_{shtl}}$  is the total heat exchanged.  $\delta \dot{Q}_{diss T} = \delta \dot{Q}_{diss f} + \delta \dot{Q}_{diss r1} + \delta \dot{Q}_{diss r2} + \delta \dot{Q}_{diss h}$  is the total energy dissipation generated by pressure drop.

There is no flow dissipation in the compression space; the mass flow of gas, Eq. (33), remains unchanged:

$$\dot{m}_c = \frac{1}{RT_{c-f}} \left( P_c \frac{d}{dt} V_c + \frac{V_c}{\gamma} \frac{dP_c}{dt} \right) \quad (50)$$

The mass flow in the different engine components is given by the expanded energy conservation equations (43)–(48):

$$\dot{m}_{c-f} = -\frac{1}{RT_{c-f}} \left( P_c \frac{dV_c}{dt} + V_c \frac{dP_c}{\gamma dt} \right) + \frac{\delta \dot{W}_{irr}}{C_p T_{c-f}} \quad (51)$$

$$\dot{m}_{f-r} = \frac{1}{C_p T_{f-r}} \left( \delta \dot{Q}_f - \delta \dot{Q}_{diss f} + C_p T_{c-f} \dot{m}_{c-f} - \frac{C_V V_f}{R} \frac{dP_c}{dt} \right) \quad (52)$$

$$\dot{m}_{r-r} = \frac{1}{C_p T_{r-r}} \left( \delta \dot{Q}_{r1} - \delta \dot{Q}_{diss r1} + C_p T_{f-r} \dot{m}_{f-r} - \frac{C_V V_{r1}}{R} \frac{dP_c}{dt} \right) \quad (53)$$

$$\dot{m}_{r-h} = \frac{1}{C_p T_{r-h}} \left( \delta \dot{Q}_{r2} - \delta \dot{Q}_{diss r2} + C_p T_{r-r} \dot{m}_{r-r} - \frac{C_V V_{r2}}{R} \frac{dP_c}{dt} \right) \quad (54)$$

$$\dot{m}_{h-d} = \frac{1}{C_p T_{h-d}} \left( \delta \dot{Q}_h - \delta \dot{Q}_{diss h} + C_p T_{r-h} \dot{m}_{r-h} - \frac{C_V V_h}{R} \frac{dP_c}{dt} \right) \quad (55)$$

The equation of continuity is recalled as follows:

$$\dot{m} = \dot{m}_E - \dot{m}_S \quad (56)$$

Successively applying Eq. (56) to the four heat exchanger cells in Fig. 2, we obtain

$$\dot{m}_f = \dot{m}_{c-f} - \dot{m}_{f-r} \quad (57)$$

$$\dot{m}_{r1} = \dot{m}_{f-r} - \dot{m}_{r-r} \quad (58)$$

$$\dot{m}_{r2} = \dot{m}_{r-r} - \dot{m}_{r-h} \quad (59)$$

$$\dot{m}_h = \dot{m}_{r-h} - \dot{m}_{h-d} \quad (60)$$

### 3. Method of solution

The model developed has been tested using data from the Stirling engine GPU-3 manufactured by General Motor. This engine has a rhombic motion transmission system, as



shown in Fig. 1. The geometrical parameters of this engine are given in Table 1. The operating conditions are as follows: working gas helium at a mean pressure of 4.13 MPa; frequency 41.72 Hz; hot space temperature  $T_{\text{Pa}h} = 977$  K; cold space temperature  $T_{\text{Pa}f} = 288$  K. The measured power output was 3958 W, at a thermal efficiency of 35%.

The independent differential equations obtained in paragraph 2, are solved simultaneously for the variables  $P_c$ ,  $m_c$ ,  $T_{r1}$ ,  $W$ , etc. The vector  $Y$  denotes the unknown functions. For example,  $Y_{Pc}$  is the system gas pressure in the compression space. The initial conditions to be satisfied are noted:  $Y(t_0) = Y_0$ .

The corresponding set of differential equations is expressed as  $dY/dt = F(t, Y)$ . The objective is to find the unknown function  $Y(t)$  which satisfies both the differential equations and the initial conditions. The numerical solution is composed of a series of short straight-line segments that approximate the true curve  $Y(t)$ . It starts from the stationary state, with  $T_c$  and  $T_d$  at  $T_{\text{Pa}f}$  and  $T_{\text{Pa}h}$  or any arbitrary initial temperature values, and goes through successive transient cycles until the values of all the state variables at the end of each cycle are equal to their values at the beginning of that cycle. The system of equations is solved numerically using the classical fourth-order Runge–Kutta method, cycle after cycle until periodic conditions are reached.

To validate the numerical method used in the computation, the results are compared with those obtained by Urieli and Berchowitz [14] for the same conditions (adiabatic models) of the GPU-3 engine data. The comparison shows a good agreement, as shown in Fig. 3.

#### 4. Dynamic model results

The model has been developed gradually, initially by neglecting the losses, then by introducing them progres-

sively. The comparison of the various models results with those obtained by Urieli and Berchowitz [14] for the same conditions of the GPU-3 engine data is shown in Table 2. The comparison shows a good agreement.

To show the effect of each loss on the engine's performances, we represented the results of the model by gradually integrating the various losses.

When all losses ( $\delta\dot{Q}_{\text{Pcdf}}$ ,  $\delta\dot{Q}_{\text{diss h}}$ ,  $\delta\dot{Q}_{\text{Pshl}}$ , etc.) are included in the model, the heat flow rate for each component versus crank angle is illustrated in Fig. 4. The corresponding average power of the engine is equal to 4.27 kW. The average heat flow generated by the heater is equal to 10.8 kW; it leads to an engine efficiency of 39.5%.

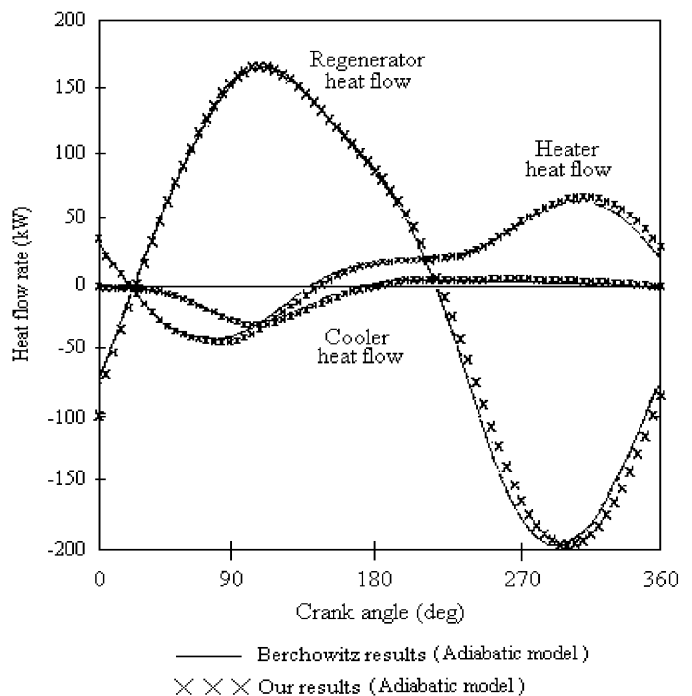


Fig. 3. Validation of the computational method.

Table 1  
Geometric parameter values of the GPU-3 Stirling engine

| Parameters                    | Values                 | Parameters           | Values                |
|-------------------------------|------------------------|----------------------|-----------------------|
| Clearance volumes             |                        | Cooler               |                       |
| Compression space             | 28.68 cm <sup>3</sup>  | Tube number/cylinder | 312                   |
| Expansion space               | 30.52 cm <sup>3</sup>  | Interns tube         | 1.08 mm               |
| Swept volumes                 |                        | Diameter             | 46.1 mm               |
| Compression space             | 113.14 cm <sup>3</sup> | Length of the tube   | 13.8 cm <sup>3</sup>  |
| Expansion space               | 120.82 cm <sup>3</sup> | Void volume          |                       |
| Exchanger piston conductivity | 15 W/m K               | Regenerator          |                       |
| Exchanger piston stroke       | 46 mm                  | Diameter             | 22.6 mm               |
| Heater                        |                        | Length               | 22.6 mm               |
| Tube number                   | 40                     | Wire diameter        | 40 μm                 |
| Tube inside diameter          | 3.02 mm                | Porosity             | 0.697                 |
| Tube length                   | 245.3 mm               | Unit number/cylinder | 8                     |
| Void volume                   | 70.88 cm <sup>3</sup>  | Thermal conductivity | 15 W/m K              |
|                               |                        | Void volume          | 50.55 cm <sup>3</sup> |

The power and the efficiency calculated by the model are very close to the power and the actual efficiency of the prototype given in the abstract, but compared to those of Martini [20], we note that they are a little different from those of Martini tests 5 and 6. This is probably due to different operating conditions and setting of equations.

The heat flow lost by internal conduction, the energy dissipation by pressure drop through the heat exchangers and the shuttle heat loss in the displacer are given in Fig. 5. The energy lost due to internal conduction is negligible in the heater and in the cooler and is about 8.5 kW in the regenerator, which represents 35% of the total energy loss. This is caused by the lengthwise temperature variation, which is very significant in the regenerator. The energy lost due to dissipation is mainly observed in the regenerator, which reaches a maximum of 3.9 kW, with an average of 935 W. In the heater and in the cooler, it is equal to 26.6 and 123 W, respectively. The average heat flow value lost by the shuttle effect is about 3.1 kW; it represents 13% of the total energy loss.

The energy lost due to external conduction in the regenerator is 27 kW, which represents 47% of the total losses, as shown in Fig. 6. It is very significant and depends mainly on the regenerator efficiency. The energy lost due to irreversibility in the compression and expansion spaces is very low [18].

## 5. Performance optimization of Stirling engines

The energy losses are mainly located in the regenerator. They are primarily due to the losses by external and internal conduction and pressure drop through the heat exchangers. The energy lost due to shuttle effect in the exchanger piston is also significant; it is about 13%. The other losses are very small [17].

The reduction of these losses improves the engine performance. Such losses depend mainly on the matrix conductivity of the regenerator, its porosity, the inlet temperature variation, the working gas mass flow rate, the

regenerator volume, and the geometrical characteristics of the displacer.

To investigate the influence of these parameters on the prototype performance, we have changed, each time, the studied parameter in the model and have kept others unchanged and equal to the prototype parameters.

### 5.1. Effect of the regenerator matrix conductivity and heat capacity

The performance of the engine depends on the conductivity and heat capacity of material constituting the regenerator matrix. Fig. 7 shows that an increase of matrix regenerator thermal conductivity leads to a reduction of

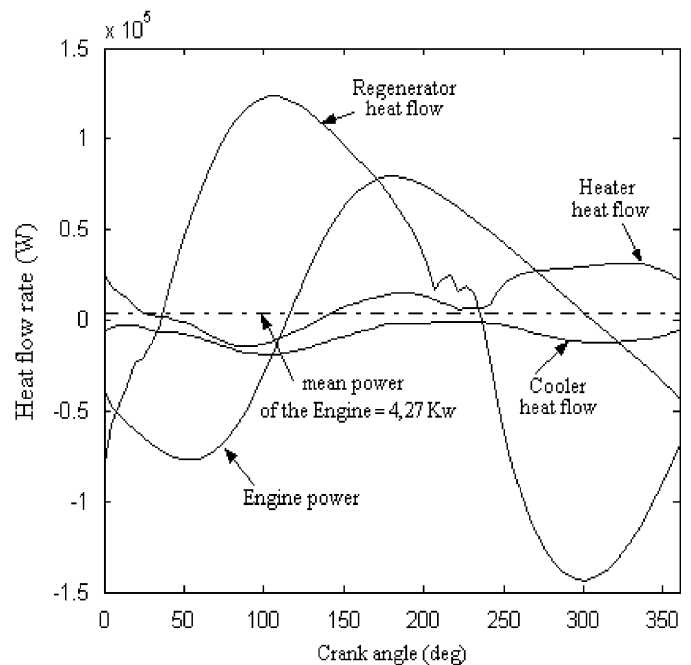


Fig. 4. Result of the dynamic model with losses.

Table 2  
Comparison of various model results

| Numerical model                                                               | Heat (J/cycle) | Indicated power output |           | Thermal efficiency (%) |
|-------------------------------------------------------------------------------|----------------|------------------------|-----------|------------------------|
|                                                                               |                | (W)                    | (J/cycle) |                        |
| Adiabatic model                                                               | 327            | 8286.7                 | 198.62    | 62.06                  |
| <i>Urielli and Berchowitz [14] adiabatic model</i>                            |                | <i>8300</i>            |           | <i>62.5</i>            |
| Dynamic model without losses                                                  | 314            | 7109.3                 | 170.4     | 54.96                  |
| <i>Urielli and Berchowitz [14] quasi-steady flow</i>                          |                | <i>7400</i>            |           | <i>53.1</i>            |
| Dynamic model with loss dissipation = (M1)                                    | 291            | 6372.4                 | 152.47    | 53.3                   |
| <i>Urielli and Berchowitz [14] quasi-steady flow (pressure drop included)</i> |                | <i>6700</i>            |           | <i>52.5</i>            |
| (M1) + Internal conduction loss = (M2)                                        | 294            | 6355.2                 | 152.32    | 52.64                  |
| M2 + External conduction loss = (M3)                                          | 314            | 6061                   | 145.27    | 46.94                  |
| M3 + Shuttle loss = (M4)                                                      | 352            | 5886.1                 | 141       | 40.66                  |
| M4 + losses by gas spring hysteresis = dynamic model with losses              | 262            | 4273                   | 99.5      | 38.49                  |
| <i>Experiment</i>                                                             | –              | <i>3958</i>            | –         | <i>35</i>              |

Urieli and Berchowitz model and experimental results are shown in italics.



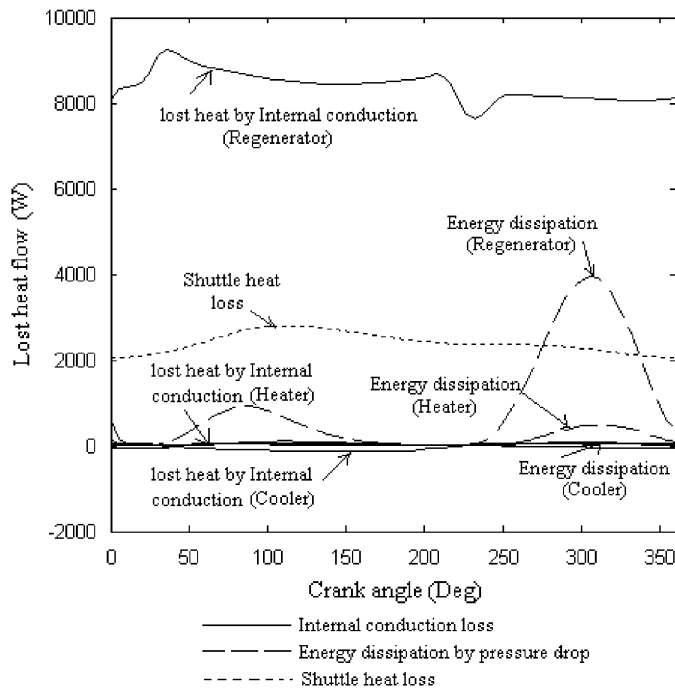


Fig. 5. Lost heat flow in the engine.

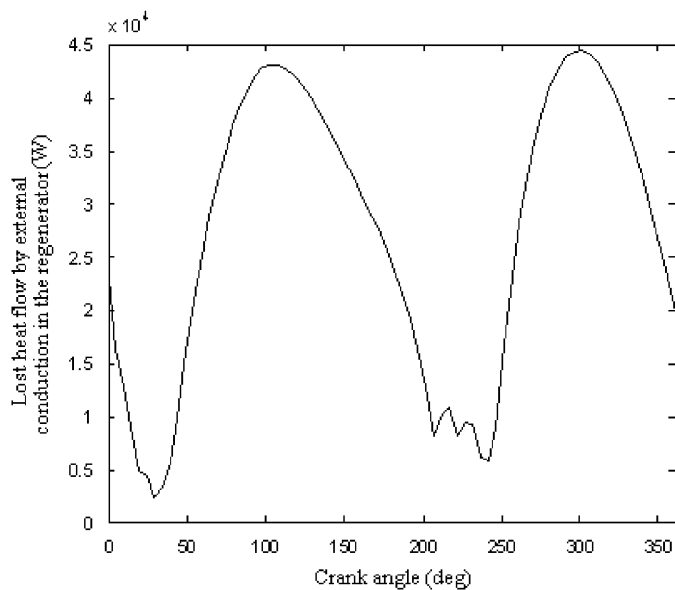


Fig. 6. Lost heat flow by external conduction in the regenerator.

performance due to the increase of internal conduction losses in the regenerator [18]. Fig. 8 shows that the engine performance improves when the heat capacity of the regenerator matrix rises.

The matrix of the regenerator can be made from several materials. The performance of the engine depends on the matrix material. To increase heat exchange of the regenerator and to reduce internal losses by conductivity, a material with high heat capacity and low conductivity must be chosen. Stainless steel and ordinary steel are the most suitable materials to make the regenerator matrix.

## 5.2. Effect of regenerator porosity

Porosity of the regenerator is an important parameter for engine performance. It affects the hydraulic diameter, dead volume, velocity of the gas, regenerator heat transfer area, and regenerator effectiveness; thus, it affects the losses by external and internal conduction and the dissipation by pressure drop [18].

The engine performance decreases when porosity increases due to an increase in the external conduction losses and a reduction of the exchanged energy between the gas and the regenerator ( $Q_r$ ), as shown in Fig. 9.

The performance decreases when porosity increases, but porosity has a low limit from which the model does not converge, due to the non-satisfaction of the boundary conditions. For the prototype studied, the calculated optimal porosity is 65.5%, as shown in Table 3.

## 5.3. Effect of regenerator temperature gradient: ( $T_{f-r} - T_{r-h}$ )

Although engine losses increase when the temperature gradient of the regenerator rises [18], the performance of the engine also increases, as shown in Fig. 10. In this event, this is due to the increase of energy exchanged between the matrix and the working fluid of the regenerator.

## 5.4. Effect of regenerator volume

To vary the regenerator volume, the diameter is fixed and the length is varied or conversely. When the regenerator diameter is fixed at 0.0226 m, length affects the performance. Although the energy exchanged increases, engine power and efficiency reach a maximum. When the length is equal to 0.01 m, power decreases quickly, as shown in Fig. 11. This can be explained by an increase of the dead volume.

When the regenerator length is constant and  $L = 0.022$  m, the performance decreases when the regenerator diameter increases, as shown in Fig. 12. The dead volume and the exchanged energy in the regenerator also decrease.

## 5.5. Effect of fluid mass

An increase of the total mass of gas in the engine leads to a rise in the density, mass flow, gas velocity, load, and function pressure. Therefore, an increase in the total mass of gas in the engine leads to more energy loss by pressure drop [18]; however, the engine power increases and the efficiency reaches a maximum of about 40% when the mass is equal to 0.8 g, as shown in Fig. 13. When the mass increases, the decrease of efficiency is due to an increase of pressure loss and the limitation of heat exchange capacity in the regenerator and the heater. The use of mass of gas equal to 1.5 g in the engine leads to an acceptable output and a higher power than in the prototype.

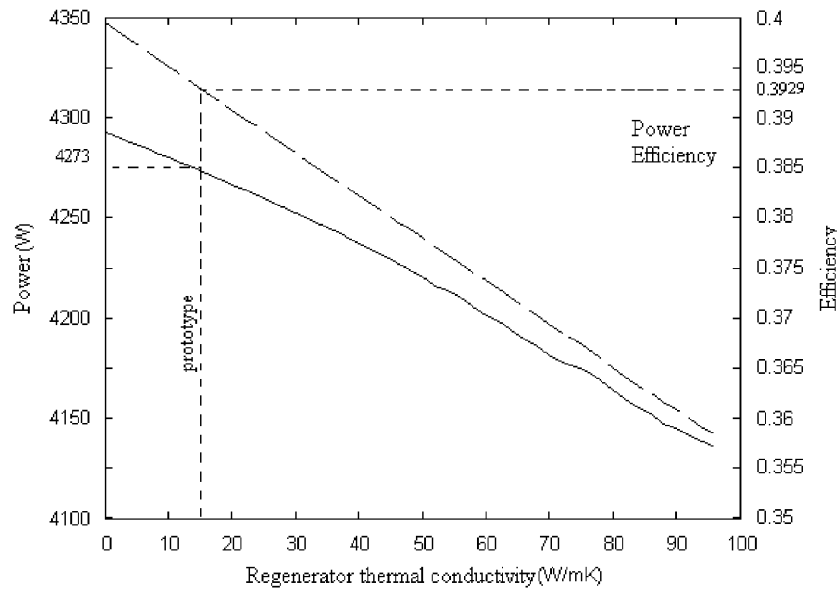


Fig. 7. Effect of regenerator thermal conductivity on performance.

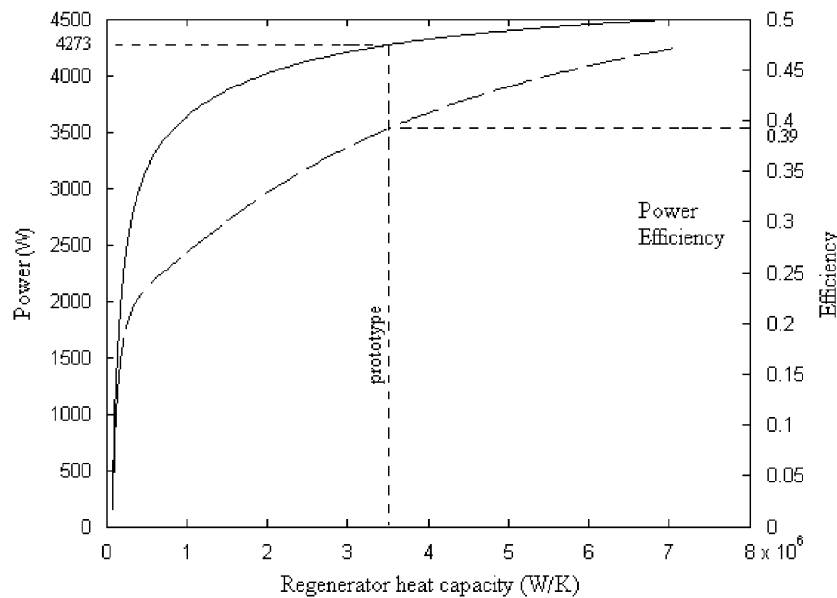


Fig. 8. Effect of regenerator heat capacity on performance.

### 5.6. Effect of expansion volume and exchanger piston conductivity

The expansion volume and the exchanger piston conductivity considerably affect the losses due to shuttle effect, which represent 13% of the engine total losses. To vary the expansion volume, we can maintain the stroke constant and vary the piston surface or conversely.

When the piston stroke is constant and equal to the prototype value of 0.046 m, the effect of the piston surface on the performance is given in Fig. 14. When the section increases, the engine power increases, but the efficiency reaches a maximum.

If the exchanger piston area is equal to  $0.0045 \text{ m}^2$ , a power higher than 5 kW and an output slightly lower than that of the prototype can be reached. When the exchanger piston area is constant and equal to the prototype value of  $0.0038 \text{ m}^2$ , the effect of stroke variation on performance is given in Fig. 15. When the stroke increases, the engine power decreases but the efficiency reaches a maximum.

The optimal performances are superior to that of the prototype. They are obtained when the area and the stroke are, respectively, equal to  $0.0038 \text{ m}^2$  and 0.042 m, which correspond to a power of 4500 W and an efficiency of 41%.

The thermal conductivity of the exchanging piston affects the engine performances considerably, as shown in

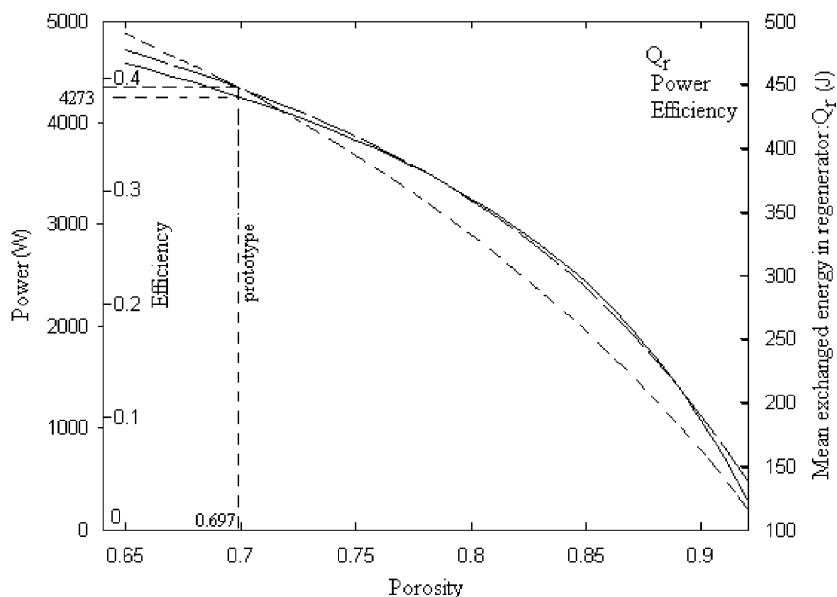


Fig. 9. Effect of regenerator porosity on performance and exchanged energy.

Table 3  
Effect of optimal parameters on engine performance

| Result optimization                     | Optimal value         | Power (W) | Efficiency (%) | Regenerator exchanged energy (J) | Average pressure function (MPa) |
|-----------------------------------------|-----------------------|-----------|----------------|----------------------------------|---------------------------------|
| Regenerator porosity                    | 65.5%                 | 4554      | 43.3           | 474.3                            | 4.72                            |
| Regenerator length (m)                  | 0.021                 | 4675      | 41             | 472.4                            | 4.79                            |
| Regenerator diameter (m)                | 0.024                 | 4765      | 39.9           | 475.8                            | 4.85                            |
| Working fluid mass (g)                  | 1.15                  | 4850      | 40.1           | 480.7                            | 4.9                             |
| Exchanger piston conductivity W/(m K)   | 1.2                   | 5079      | 50.9           | 479.9                            | 4.95                            |
| Exchanger piston area (m <sup>2</sup> ) | $3.86 \times 10^{-3}$ | 5183      | 50.97          | 483.7                            | 4.92                            |
| Exchanger piston stroke (m)             | 0.047                 | 5106      | 51.1           | 472.7                            | 4.95                            |
| Model result before optimization        |                       | 4273      | 38.5           | 448.7                            | 4.67                            |
| Experiment result of prototype          |                       | 3958      | 35             |                                  | 4.13                            |

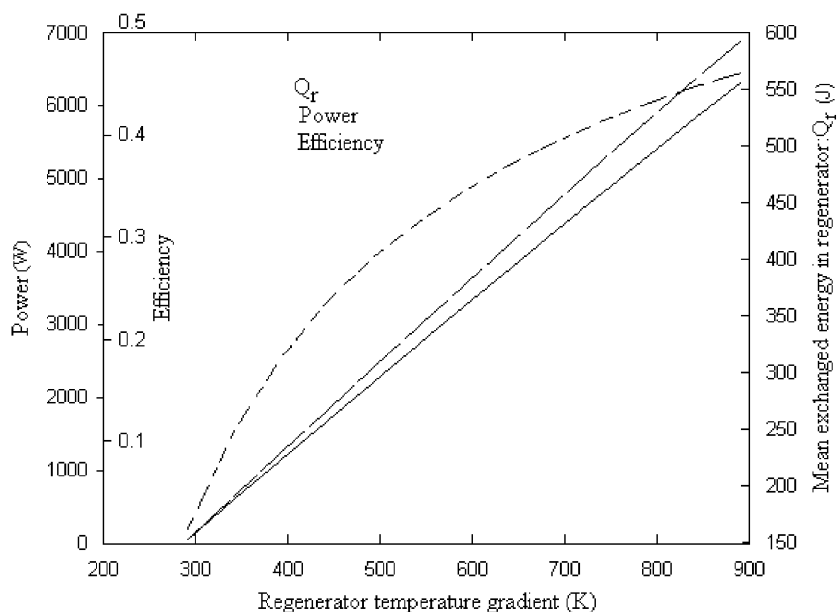


Fig. 10. Effect of regenerator temperature gradient on performance and exchanged energy.

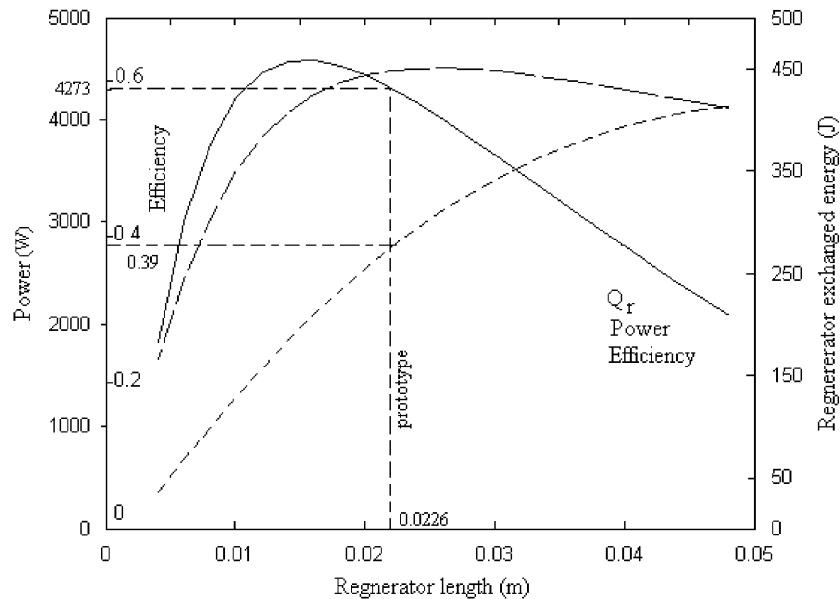


Fig. 11. Effect of regenerator length on performance and exchanged energy.

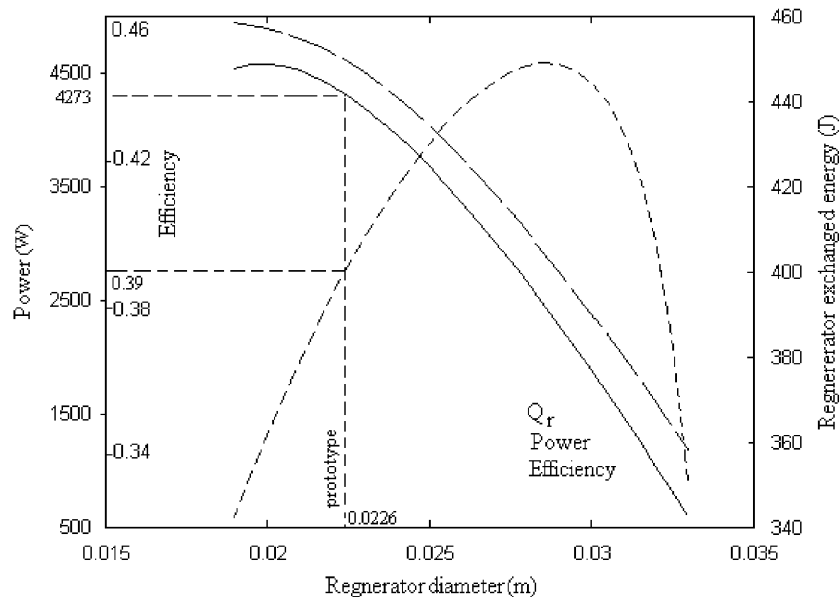


Fig. 12. Effect of regenerator diameter on performance and exchanged energy.

Fig. 16. Weak conductivity reduces the losses by shuttle effect and consequently increases the engine power and efficiency.

## 6. Design optimization of Stirling engine parameters

Based on the reduction of losses and optimization of performance, the optimization consists first in determining the optimal value of each parameter when the other parameters are equal to the prototype parameters, second in gradually replacing the parameters of the prototype by their optimal values. The results are presented in Table 3.

The optimal parameters for the design are found as follows.

For the first line of Table 2, the prototype porosity (0.697) is the only parameter replaced by the optimal porosity (0.655) in the model. The power and efficiency are improved, but the average pressure increases slightly. Therefore, the optimal porosity is of the model is maintained and the regenerator optimal length (0.021 m) is to be searched. By introducing this length into the model, the power improves and the efficiency remains acceptable. Then, the optimal diameter (0.024 m) is used; the power and the exchanged energy in the regenerator increase but

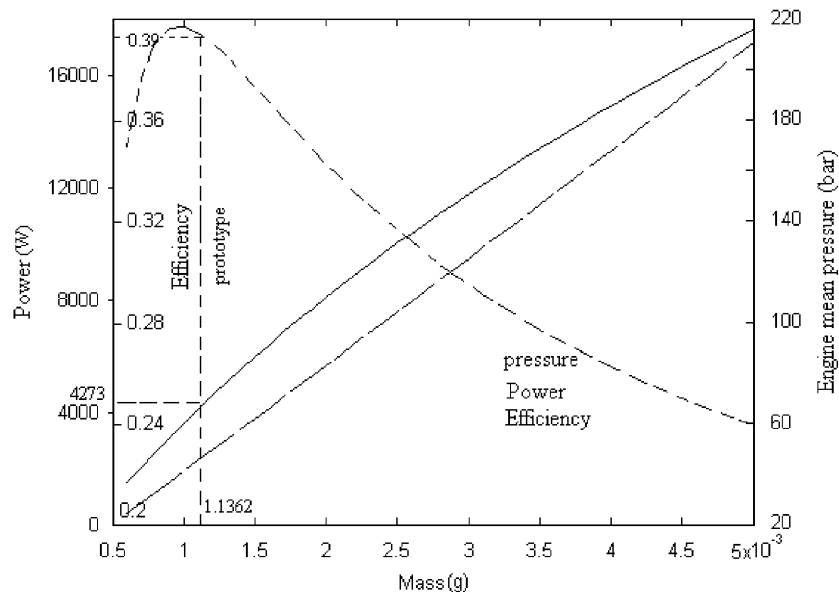


Fig. 13. Effect of fluid mass on performance and engine mean pressure.

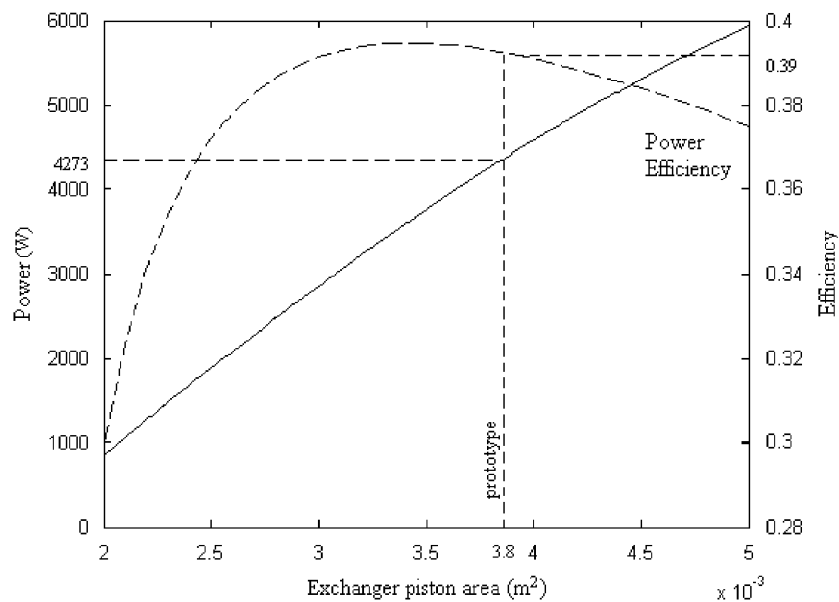


Fig. 14. Effect of exchanger piston area on engine performance.

the efficiency decreases slightly. By replacing the working fluid mass by the calculated optimal value (1.15 g) in the model, the power and the efficiency increase but the calculated engine average pressure remains close to the initial value.

The exchanger piston conductivity influences the loss by shuttle effect, its reduction leads to a remarkable increase in the performances. By introducing optimal values of the area and the stroke of the exchanger piston in the model, the prototype performance clearly improves.

## 7. Conclusion

The theoretical Stirling cycle has a high theoretical efficiency; however, the constructed prototypes have a low

output because of considerable losses in the regenerator and the exchanger piston. This is primarily due to losses by external and internal conduction, pressure drops in the regenerator, and by shuttle effect in the exchanger piston. These losses depend on the geometrical and physical parameters of the prototype design.

An optimization of these parameters has been carried out using the GPU-3 engine data, and has led to a reduction of losses and to a notable improvement in the engine performance. We first applied the parameters of this prototype on the developed model; the results were very close to the experimental data. Then, we studied the influence of each geometrical and physical parameter on the engine performance and the exchange energy of the regenerator. The reduction of matrix porosity and

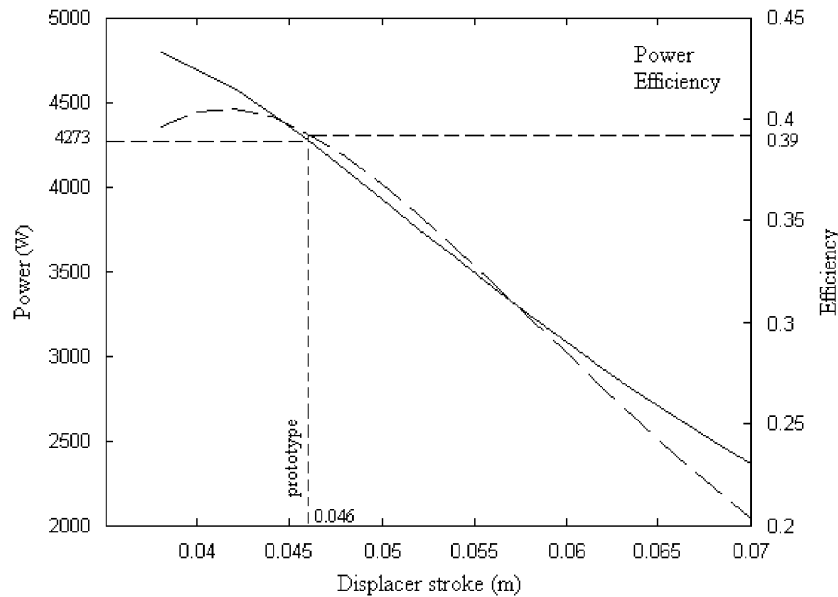


Fig. 15. Effect of exchanger piston stroke on engine performance.

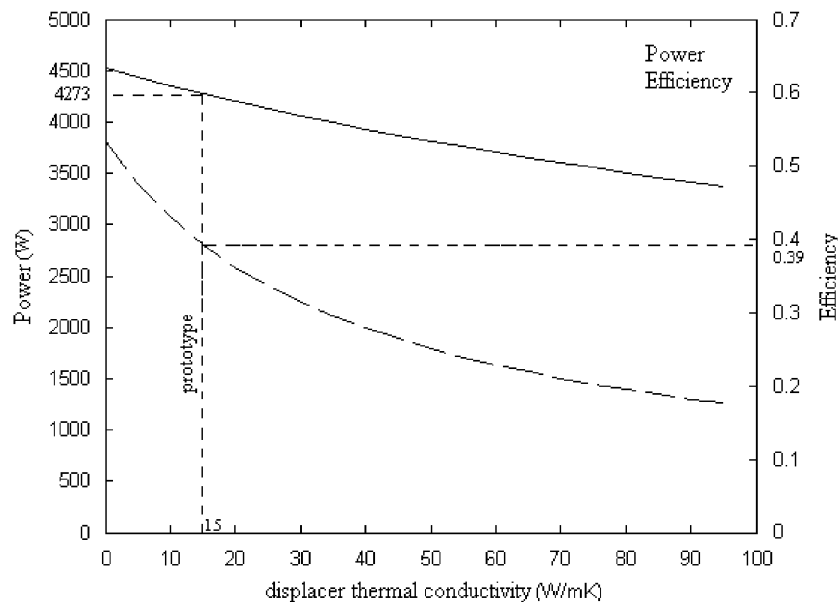


Fig. 16. Effect of displacer thermal conductivity on performance.

conductivity of the regenerator increases the performance. A rise of the total gas mass leads to an increase of the engine power and working pressure. However, the efficiency reaches a maximum. When the displacer section increases and the piston stroke decreases, the engine power increases, and the efficiency reaches a maximum. A low conductivity of the exchanger piston reduces the losses by shuttle effect and consequently increases the engine power and efficiency.

Finally, we optimized these parameters gradually by introducing them into the model and seeking each time the optimum value. Although the actual performance of the used prototype is relatively high, results have improved considerably. The efficiency increased from 39% to 51%,

the power rose approximately 20%, and the average pressure slightly increased.

Applying the proposed approach to prototypes with low performance or to newly designed engines will lead to the determination of their optimal design parameters and consequently to a higher performance.

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